CONCURSUL NAȚIONAL DE MATEMATICĂ
„TEHNICI MATEMATICE”-editia a XIX-a
Etapa judeteană 23.02.2024
Clasa a X -a Matematică *M_tehnologic*

Barem de corectare

Subiectul I (30p)

a) $D: \begin{cases} x+5 \geq 0 \\ x+2 \geq 0 \\ x^2+7x+10 \geq 0 \end{cases} => D = [-2, +\infty)$.....1p

$\frac{3}{\sqrt{x+5}+\sqrt{x+2}} \cdot \left(\sqrt{x+5} \cdot \sqrt{x+2} + 1\right) = 3 \ |: 3$.....1p

$\frac{\sqrt{x+5} \cdot \sqrt{x+2}+1}{\sqrt{x+5}+\sqrt{x+2}} = 1$.....1p

$\sqrt{x+5} + \sqrt{x+2} = \sqrt{x+5} \cdot \sqrt{x+2} + 1$.....1p

$\sqrt{x+5} + \sqrt{x+2} - \sqrt{x+5} \cdot \sqrt{x+2} - 1 = 0$.....1p

$\sqrt{x+5} \cdot \left(1 - \sqrt{x+2}\right) - \left(1 - \sqrt{x+2}\right) = 0$.....1p

$\left(1 - \sqrt{x+2}\right) \cdot \left(\sqrt{x+5} - 1\right) = 0 \ | \cdot (-1)$.....1p

$\left(\sqrt{x+2} - 1\right) \cdot \left(\sqrt{x+5} - 1\right) = 0$ 1p

I) $\sqrt{x+2} - 1 = 0 <=> \sqrt{x+2} = 1 <=> x+2 = 1 <=> x = -1 \in D$ 1p

II) $\sqrt{x+5} - 1 = 0 <=> \sqrt{x+5} = 1 <=> x+5 = 1 <=> x = -4 \notin D$ 1p

b) $36^a = 4 => \log_{36} 36^a = \log_{36} 4 => a = \log_{36} 4$ 2p

$36^b = 3 => \log_{36} 36^b = \log_{36} 3 => b = \log_{36} 3$ 2p

$\left(\sqrt[1012]{2401}\right)^{\frac{1-\log_{36} 4-\log_{36} 3}{1-\log_{36} 4}} = \left(\sqrt[1012]{2401}\right)^{\frac{\log_{36} 36-\log_{36} 12}{\log_{36} 36-\log_{36} 4}} = \left(\sqrt[1012]{2401}\right)^{\frac{\log_{36} 3}{\log_{36} 9}} =$.....2p

$= \left(\sqrt[1012]{2401}\right)^{\frac{1}{2}} = \left(2401^{\frac{1}{1012}}\right)^{\frac{1}{2}} = (2401)^{\frac{1}{2024}} = (7^4)^{\frac{1}{2024}} = 7^{\frac{4}{2024}} = 7^{\frac{1}{506}} = \sqrt[506]{7}$.....4p

c) Fie: $t = \log_{2023} 2024, t > 0$.....2p

Dacă: $a, b \in (1, \infty) => \log_a b > 0$ 2p

$t + \frac{1}{t} > 2 <=> t^2 - 2t + 1 > 0 <=> (t-1)^2 > 0 \ (A)$.....6p

Subiectul II (30p)

a) Notăm: $(7+4\sqrt{3})^x = t, t > 0 => t + \frac{1}{t} = 14$.....2p

$<=> t^2 - 14t + 1 = 0$.....2p

$\Delta = 196 - 4 = 192, \Delta = 8\sqrt{3}$.....2p

$t_1 = \frac{14+8\sqrt{3}}{2} = 7 + 4\sqrt{3}, t_2 = \frac{14-8\sqrt{3}}{2} = 7 - 4\sqrt{3}$.....2p

I) $(7+4\sqrt{3})^x = 7 + 4\sqrt{3} => x = 1$.....1p

II) $(7+4\sqrt{3})^x = 7 - 4\sqrt{3} => x = -1$.....1p

b) $5^x = 10 - 5^{-x}$

$5^x + 5^{-x} - 10 = 0 => 5^x - 10 + \frac{1}{5^x} = 0 => 5^{2x} - 10 \cdot 5^x + 1 = 0$.....2p

$5^{2x} - 2 \cdot 5 \cdot 5^x + 1 = 0 => 5^{2x} - 2 \cdot 5 \cdot 5^x + 25 - 24 = 0$.....2p

$(5^x-5)^2-24=0 => (5^x-5)^2=24$.....2p
Dar $5^x-5=4^y$
$(4^y)^2=24 => 4^{2y}=24$.....2p
$(4^{2y})^{\frac{1}{y}}=(24)^{\frac{1}{y}} => (24)^{\frac{1}{y}}=4^2=16$.....2p

c) $3^{2x}+3\cdot 3^x-4=0$ 2p
Notăm $3^x=t, t>0$ 2p
$t^2+3\cdot t-4=0$.....2p
$t_1=\frac{-3+5}{2}=1, t_2=\frac{-3-5}{2}=-4<0$ 2p
$3^x=1 => 3^x=3^0 => x=0$ 2p

Subiectul III (30p)
a) f injectivă: $(\forall)x_1,x_2\in\mathbb{R}, x_1\neq x_2$ avem $f(x_1)\neq f(x_2)$.....1p
$f(x_1)=f(x_2) <=> 2x_1+3=2x_2+3 <=> x_1=x_2$ (1).....1p
f surjectivă: $(\forall)y\in\mathbb{R}, (\exists)x\in\mathbb{R}$ astfel încât , $f(x)=y$.....1p
$2x+3=y <=> 2x=y-3 <=> x=\frac{y-3}{2}\in\mathbb{R}\ (A)$ (2) 1p
Din (1) și (2) => f − bijectivă => f – inversabilă.....1p
$f^{-1}:\mathbb{R}\to\mathbb{R}, f^{-1}(x)=\frac{x-3}{2}$.....1p
$f(x)\cdot f^{-1}(x)=\frac{11}{2} <=> (2x+3)\cdot\frac{x-3}{2}=\frac{11}{2} <=> (2x+3)\cdot(x-3)=11$ 2p
$<=> 2x^2-3x-20=0 => x_1=4; x_2=-\frac{5}{2}$.....2p

b) Cum: $z=\frac{-1-i\sqrt{3}}{2} => z^2=\frac{-1+i\sqrt{3}}{2}$ 4p
$1+z+z^2=1+\frac{-1-i\sqrt{3}}{2}+\frac{-1+i\sqrt{3}}{2}=$ 3p
$=\frac{2-1-i\sqrt{3}-1+i\sqrt{3}}{2}=0$ 3p

c) Fie: $z=a+ib; a,b\in\mathbb{R}; \bar{z}=a-ib$
$\frac{2(a+ib)i+3-4i}{1+2i+(a-ib)i}=\frac{9}{4}$.....1p
$\frac{2ai+2bi^2+3-4i}{1+2i+ai-bi^2}=\frac{9}{4}$.....1p
$\frac{2ai-2b+3-4i}{1+2i+ai+b}=\frac{9}{4}$.....1p
$9(1+2i+ai+b)=4(2ai-2b+3-4i)$.....1p
$9+18i+9ai+9b=8ai-8b+12-16i$.....1p
$(9+9b)+i(18+9a)=(-8b+12)+i(8a-16)$.....1p
$\begin{cases}9+9b=-8b+12\\18+9a=8a-16\end{cases}$.....1p
$\begin{cases}9b+8b=12-9\\9a-8a=-16-18\end{cases}$.....1p
$\begin{cases}17b=3\\a=-34\end{cases} <=> \begin{cases}b=\frac{3}{17}\\a=-34\end{cases} <=> z=-34+\frac{3}{17}i$.....2p